

# Universality of the rho-meson coupling in effective field theory

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It is shown that both the universal coupling of the  $\rho$  meson and the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin expression for the magnitude of its coupling constant follow from the requirement that chiral perturbation theory of pions, nucleons, and  $\rho$  mesons is a consistent effective field theory. The prerequisite of the derivation is that all ultraviolet divergences can be absorbed in the redefinition of fields and the available parameters of the most general effective Lagrangian.

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Already in the 1960s, vector mesons were discussed in the framework of phenomenological low-energy chiral Lagrangians [1, 2, 3]. For the details of the construction of chirally invariant effective Lagrangians describing the interaction of vector mesons with pseudoscalars and baryons see, e.g., Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12]. In Ref. [12], different formulations of vector-meson effective theories were shown to be equivalent. In the massive Yang-Mills approach (for a review see, e.g., Ref. [4]) vector mesons are treated as gauge bosons of local chiral symmetry (with symmetry breaking mass terms added by hand). This scenario implies that the  $\rho$  meson couples universally, i.e. with the same strength, to fermions and pseudoscalars. In the so-called hidden chiral symmetry approach (see, e.g., Refs. [4, 5, 6])  $\rho$ -coupling universality is obtained only with a specific choice of a free parameter of the Lagrangian. Although the hypotheses of dynamical bosons of both approximate and hidden local chiral symmetry are attractive, there is, as was emphasized in Ref. [8], no proof for the existence of such gauge bosons of local chiral symmetry in QCD.

Besides the construction of the effective Lagrangian, a consistent effective field theory (EFT) program requires a systematic power counting which allows one to organize the perturbation series. For example, within the extended on-mass-shell renormalization scheme of Ref. [13] it is possible to consistently include virtual (axial-) vector mesons in a manifestly Lorentz-invariant formulation of the EFT [14], provided they appear only as internal lines in Feynman diagrams involving soft external pions and nucleons with small three-momenta. Moreover, a consistent power counting also exists within the reformulated version [15] of the infrared renormalization of Becher and Leutwyler [16].

In this letter we consider the effective Lagrangian of Ref. [3] describing the interaction among  $\rho$  mesons, pions, and nucleons. In principle, the Lagrangian contains *all* interaction terms which respect Lorentz invariance, the discrete symmetries, and chiral symmetry. As was stressed in Ref. [3], the equality of the  $\rho\pi\pi$  and the  $\rho NN$  coupling constants *does not* follow as a consequence of the symmetries of the Lagrangian. Below we perform

a one-loop order analysis of the nucleon and  $\rho$ -meson self-energies as well as the  $\rho\rho\rho$  and  $\rho NN$  vertex functions. In accordance with the general principles of effective field theory [17], we require that all ultra-violet (UV) divergences can be absorbed into the redefinition of fields, masses, and coupling constants, as long as one includes every one of the infinite number of interactions allowed by symmetries [18]. The renormalization procedure imposes consistency conditions among the (renormalized) parameters of the Lagrangian. In our case, both the universal coupling of the  $\rho$  meson as well as the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSFR) value of the  $\rho$ -meson coupling constant [19, 20] turn out to be *consequences* of the self-consistency conditions imposed by the EFT approach.

We start from the chirally invariant effective Lagrangian, including vector mesons, in the form given by Weinberg [3],

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu \pi_0^a \partial^\mu \pi_0^a - \frac{M_0^2}{2} \pi_0^a \pi_0^a \\ & + \bar{\Psi}_0 (i \gamma^\mu \partial_\mu - m_0) \Psi_0 \\ & - \frac{1}{4} F_{\mu\nu 0}^a F_0^{a\mu\nu} + \frac{1}{2} M_{\rho 0}^2 \rho_{\mu 0}^a \rho_0^{a\mu} \\ & + g_{\rho\pi\pi 0} \epsilon^{abc} \pi_0^a \partial_\mu \pi_0^b \rho_0^{c\mu} \\ & + g_{\rho NN 0} \bar{\Psi}_0 \gamma^\mu \frac{\tau^a}{2} \Psi_0 \rho_{\mu 0}^a + \mathcal{L}_1, \end{aligned} \quad (1)$$

where  $\pi_0^a$  and  $\rho_{\mu 0}^a$  are isospin triplets of pion and  $\rho$ -meson fields with masses  $M_0$  and  $M_{\rho 0}$ , respectively, and  $\Psi_0$  is an isospin doublet of nucleon fields with mass  $m_0$  [21]. The field strengths are defined as  $F_{\mu\nu 0}^a = \partial_\mu \rho_{\nu 0}^a - \partial_\nu \rho_{\mu 0}^a + g_0 \epsilon^{abc} \rho_{\mu 0}^b \rho_{\nu 0}^c$ , where  $g_{\rho NN 0} \equiv g_0$  follows from chiral symmetry [3]. Finally,  $\mathcal{L}_1$  contains an infinite number of terms allowed by the symmetries of the theory [17, 18]. In Eq. (1) the subscripts 0 stand for bare quantities. In principle, a  $\rho\rho\pi\pi$  interaction would also have a dimensionless coupling constant, but it is not included in the Lagrangian of Eq. (1), because it is not consistent with chiral symmetry in the present parametrization of fields [3].

In order to establish relations among the renormalized coupling constants pertaining to the Lagrangian of Eq. (1), we analyze the renormalization of the coupling constant  $g_0$  using dimensional regularization in combination with the minimal subtraction (MS) scheme (for a definition see, e.g., Ref. [22]). However, our findings below do not depend on the choice of a specific renormalization scheme. For that purpose we rewrite the Lagrangian in terms of the renormalized fields  $\pi^a$ ,  $\Psi$ , and  $\rho_\mu^a$  as well as the renormalized parameters  $g$ ,  $M$ ,  $m$ , and  $M_\rho$ ,

$$\begin{aligned} \pi_0^a &= \sqrt{Z_\pi} \pi^a, & Z_\pi &= 1 + \delta Z_\pi, \\ \Psi_0 &= \sqrt{Z_\Psi} \Psi, & Z_\Psi &= 1 + \delta Z_\Psi \\ \rho_0^{a\mu} &= \sqrt{Z_\rho} \rho^{a\mu}, & Z_\rho &= 1 + \delta Z_\rho, \\ g_0 &= g + \delta g, \\ g_{\rho\pi\pi 0} &= g_{\rho\pi\pi} + \delta g_{\rho\pi\pi}, \\ M_0^2(1 + \delta Z_\pi) &= M^2 + \delta M^2, \\ m_0(1 + \delta Z_\Psi) &= m + \delta m, \\ M_{\rho 0}^2(1 + \delta Z_\rho) &= M_\rho^2 + \delta M_\rho^2. \end{aligned} \quad (2)$$

Using Eq. (2), we re-express the Lagrangian of Eq. (1) as [22]

$$\mathcal{L} = \mathcal{L}_{\text{basic}} + \mathcal{L}_{\text{ct}} + \tilde{\mathcal{L}}_1,$$

with the basic Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{basic}} &= \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a - \frac{M^2}{2} \pi^a \pi^a \\ &+ \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi \\ &- \frac{1}{4} A_{\mu\nu}^a A^{\mu\nu} + \frac{1}{2} M_\rho^2 \rho_\mu^a \rho^{a\mu} \\ &+ g_{\rho\pi\pi} \epsilon^{abc} \pi^a \partial_\mu \pi^b \rho^{c\mu} \\ &- g \epsilon^{abc} \partial_\mu \rho_\nu^a \rho^{b\mu} \rho^{c\nu} \\ &- \frac{1}{4} g^2 \epsilon^{abc} \epsilon^{ade} \rho_\mu^b \rho_\nu^c \rho^{d\mu} \rho^{e\nu} \\ &+ g \bar{\Psi} \gamma^\mu \frac{\tau^a}{2} \Psi \rho_\mu^a, \end{aligned} \quad (3)$$

the counterterm Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{ct}} &= -\frac{\delta Z_\rho}{4} A_{\mu\nu}^a A^{\mu\nu} \\ &+ \left[ \delta g_{\rho\pi\pi} + g_{\rho\pi\pi} \left( \frac{\delta Z_\rho}{2} + \delta Z_\pi \right) \right] \epsilon^{abc} \pi^a \partial_\mu \pi^b \rho^{c\mu} \\ &- \left( \delta g + \frac{3}{2} g \delta Z_\rho \right) \epsilon^{abc} \partial_\mu \rho_\nu^a \rho^{b\mu} \rho^{c\nu} \\ &+ \left[ \delta g + g \left( \frac{\delta Z_\rho}{2} + \delta Z_\Psi \right) \right] \bar{\Psi} \gamma^\mu \frac{\tau^a}{2} \Psi \rho_\mu^a, \end{aligned} \quad (4)$$

and the residual piece  $\tilde{\mathcal{L}}_1$ . In Eqs. (3) and (4), we defined  $A_{\mu\nu}^a \equiv \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a$ , and in  $\mathcal{L}_{\text{ct}}$  we only displayed those counterterms explicitly which are relevant for the subsequent discussion. All remaining counterterms are included in  $\tilde{\mathcal{L}}_1$ .

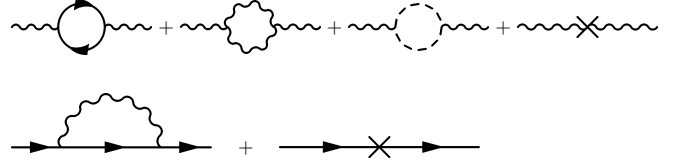


FIG. 1: Self-energy diagrams. Solid, wiggly, and dashed lines correspond to nucleons,  $\rho$  mesons, and pions, respectively.

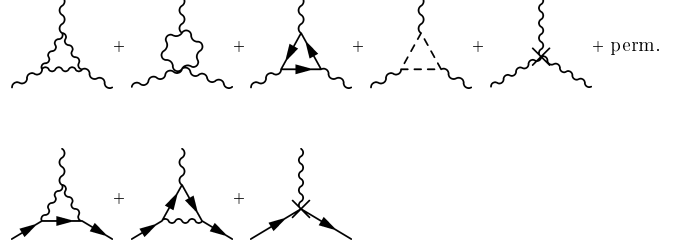


FIG. 2: Vertex diagrams. Solid, wiggly, and dashed lines correspond to nucleons,  $\rho$  mesons, and pions, respectively.

In the following, we will first derive the universality of the  $\rho$  coupling, i.e.,  $g_{\rho\pi\pi} = g$ . To that end, let us consider the  $\rho\rho\rho$  and  $\rho\Psi\Psi$  vertex functions (amputated Green's functions) at one-loop order. At that order there are two types of contributions: the genuine one-loop diagrams with basic interaction vertices and the tree-level diagrams with one-loop order counterterms. We will make use of the fact that the combination of these two types of contributions must lead to UV finite results.

Up to and including one-loop order, the renormalization constant of the  $\rho$ -meson field receives contributions of the form

$$\frac{1}{2} \delta Z_\rho = z_1 g^2 + z_2 g_{\rho\pi\pi}^2, \quad (5)$$

where the  $z_1$  term corresponds to  $\rho$ -meson and fermion loop contributions, and the  $z_2$  term to the pion-loop contribution, respectively (see Fig. 1). We combine the contribution of the counterterm diagram generated by the  $\rho\rho\rho$  term in the third line of Eq. (4) with the divergent parts of the one-loop contributions to the  $\rho\rho\rho$  vertex given in Fig. 2. Requiring that the result vanishes, we obtain

$$\Gamma_1 g^3 + \Gamma_2 g_{\rho\pi\pi}^3 - \delta g - \frac{3}{2} g \delta Z_\rho = 0, \quad (6)$$

where the  $\Gamma_1$  term is generated by  $\rho$ -meson and fermion loops, and the  $\Gamma_2$  term by the pion loop, respectively. Substituting Eq. (5) in Eq. (6) and solving for  $\delta g$  we obtain

$$\delta g = \Gamma_1 g^3 + \Gamma_2 g_{\rho\pi\pi}^3 - 3z_1 g^3 - 3z_2 g g_{\rho\pi\pi}^2. \quad (7)$$

Next, we go through analogous arguments for the  $\rho NN$  term. First, the one-loop  $\rho$ -meson contribution to the

renormalization constant of the  $\Psi$  field, in terms of the renormalized couplings, reads

$$\frac{1}{2}\delta Z_\Psi = w_1 g^2. \quad (8)$$

Again, we combine the divergent parts of the one-loop contributions to the  $\rho\bar{\Psi}\Psi$  vertex given in Fig. 2 with the contribution of the counterterm diagram generated by the  $\rho\bar{\Psi}\Psi$  term in the last line of Eq. (4) and require the result to vanish:

$$D_1 g^3 + \delta g + g \frac{1}{2} \delta Z_\rho + g \delta Z_\Psi = 0. \quad (9)$$

Substituting Eq. (8) in Eq. (9) and solving for  $\delta g$  we obtain

$$\delta g = -D_1 g^3 - (z_1 + 2w_1) g^3 - z_2 g g_{\rho\pi\pi}^2. \quad (10)$$

In order to have a self-consistent theory, the two expressions for  $\delta g$  given by Eqs. (7) and (10) must coincide. Calculating  $z_1$ ,  $z_2$ ,  $w_1$ ,  $\Gamma_1$ ,  $\Gamma_2$ , and  $D_1$  explicitly [23] and comparing Eqs. (7) and (10) we obtain:

$$g_{\rho\pi\pi}^3 = g g_{\rho\pi\pi}^2. \quad (11)$$

Equation (11) has a trivial solution  $g_{\rho\pi\pi} = 0$ , which corresponds to the EFT without pions, and the non-trivial solution

$$g_{\rho\pi\pi} = g. \quad (12)$$

In other words, from the EFT standpoint the universality of the  $\rho$  coupling,  $g_{\rho\pi\pi} = g$ , is a *consequence* of the consistency conditions imposed by the requirement of perturbative renormalizability [24]. From this point of view, universality neither appears to be something which has to be postulated [19], nor is it obtained as the result of a dynamical principle such as vector-meson dominance [20].

As has been pointed out in, e.g., Refs. [3, 7], chiral symmetry specifies the combination of the vector-meson mass term and the  $\rho\pi\pi$  coupling that should appear in the Lagrangian. This combination reads

$$\frac{M_\rho^2}{2} \rho_{\mu 0}^a \rho_0^{a\mu} - \frac{1}{2} M_0^2 g_0^{-1} F_0^{-2} \epsilon^{abc} \pi_0^a \partial_\mu \pi_0^b \rho_0^{c\mu}, \quad (13)$$

where  $F_0$  denotes the bare pion-decay constant in the chiral limit. Comparing Eq. (13) with the Lagrangian of Eq. (1) it follows that, in order to retain chiral symmetry, the relation

$$g_{\rho\pi\pi 0} = \frac{M_{\rho 0}^2}{2g_0 F_0^2} \quad (14)$$

should hold. Taking Eq. (12) into account and performing a loop expansion of bare quantities,  $g_0 = g + O(\hbar)$

etc., we obtain from Eq. (14), in terms of renormalized quantities,

$$g^2 = \frac{M_\rho^2}{2F^2}. \quad (15)$$

Equation (15) is the well-known Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation [19, 20] which was originally derived by combining current algebra with soft-pion techniques and either assuming universality [19] or assuming vector-meson dominance in conjunction with the conserved vector current hypothesis [20]. In the EFT framework, this relation is a natural consequence of chiral symmetry and the consistency conditions which are imposed on the parameters of the effective Lagrangian by the requirement of renormalizability.

We would like to emphasize that our analysis of the renormalizability of the considered EFT is only partial (the complete analysis is beyond the scope of this letter). For example, in addition to the divergences explicitly analyzed here, the considered diagrams contain divergences requiring counterterms which have a non-renormalizable structure (in the traditional sense). For the consistent EFT it should be possible to absorb all these divergences in the redefinition of the parameters of  $\mathcal{L}_1$ . Furthermore, the applied dimensional regularization implicitly subtracts all power-law divergences. In a complete analysis it would be necessary to show that these divergences can be absorbed in the redefinition of the parameters and fields of the most general effective Lagrangian. Although we believe that, in the sense of renormalizability, there exists a consistent EFT of nucleons, pions and vector-mesons, to the best of our knowledge this never has been proven. Therefore, strictly speaking, our results should be interpreted that, given the existence of a consistent EFT, universality and the KSRF relation necessarily hold.

To summarize, we have considered the most general effective Lagrangian for the interaction of  $\rho$  mesons with pions and nucleons, respecting Lorentz invariance, the discrete symmetries, and chiral symmetry. While usually the universal  $\rho$  coupling is taken as an additional *assumption*, in the framework of EFT it has a deep foundation, namely the consistency of EFT with respect to renormalization. In addition, combining chiral symmetry and the consistency of EFT naturally generates the KSRF relation among the renormalized  $\rho$ -coupling constant,  $\rho$ -meson mass, and pion-decay constant.

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